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Optimal Crash Pulse for Minimization of Peak Occupant Deceleration in Frontal Impact

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ABSTRACT

In automobile frontal impact, for given restraint characteristics and prescribed impact speed and crash deformation, what is the optimal vehicle crash pulse that produces the lowest peak occupant deceleration? In this paper, based on a lumped-parameter model of the occupant-vehicle system, the problem is treated as a best disturbance problem of the optimal protection from impact. The optimum kinematics of the occupant in frontal impact is found. For linear restraint characteristics, the theoretical optimal crash pulse is obtained, and the relation of the peak occupant deceleration to the impact speed, crash deformation, and vehicle interior rattlespace is established. The optimal crash pulses for passive, active, and pre-acting restraint systems are discussed. Numerical optimization is formulated to find the optimal crash pulse for general restraint systems. The occupant responses under the constant-deceleration crash pulse and the half-sine crash pulse are analyzed for linear restraint systems. Comparisons are made between the optimal crash pulse and these two "non-optimal" crash pulses.

INTRODUCTION

The structural crashworthiness plays important roles in the prevention and reduction of occupant injuries in automobile crashes. To attain the optimal crashworthiness of an automobile structure, optimization is an effective tool and becomes a common practice in the vehicle design. Optimization can now be conducted in a virtual environment based on computational modeling, simulation, and analysis of a system.

The optimization problem of the vehicle crashworthiness can be dealt with analytically with two approaches, direct and indirect [1]. The direct approach uses physical parameters of the vehicle structure as the design variables in optimization. The indirect approach handles the issue by a two-step strategy: first, for the minimization of the occupant injuries, find the optimal response for the vehicle structure; then, solve an inverse design problem for the vehicle structure according to its optimal response or use the optimal response as a guideline for the vehicle structure design.

The study in this paper will take the indirect approach and deal only with the problem in the first step for frontal impact, i.e., the optimal vehicle crash response. In particular, the issue to be addressed is: in frontal impact, for given restraint characteristics and prescribed impact speed and crash deformation of the vehicle, what is the optimal vehicle crash pulse for mitigating occupant injuries?

A number of studies have been performed on the vehicle crash pulse in the last three decades or so. Several late studies were particularly concerned with the problem of the optimal crash pulse [1-4]. In [2], based on a one-mass model and a rigid multi-body model, the vehicle crash deceleration curve was divided into finite segments and their effects on the occupant's injury were investigated quantitatively using a sensitivity analysis. The optimal deceleration curve for minimizing the occupant's injury was obtained by iterative calculations. In [3], based on a two-mass one-dimensional model, pure rectilinear form is assumed as the ideal pulse for the occupant deceleration. Three functions were formed to approximate the rising edge of the rectilinear form and corresponding initial parts of the vehicle crash pulse were derived. In [1] based on a spring-mass model, the problem of the optimal crash pulse is addressed by applying an energy relationship between the vehicle crash pulse and the occupant deceleration in a domain of the relative motion between the vehicle and the occupant. It was found that for prescribed impact velocity and vehicle crash deformation, the optimal pulse is one that consists of an impulse, a subsequent zero-acceleration period, and finally a constant level period. In [4], the problem was also described by a one-mass model, the crash pulse was discretized with respect to the crash deformation, and a solution procedure using numerical optimization was proposed to find the optimal crash pulse.

In this paper, the problem of the optimal vehicle crash pulse is treated as a best disturbance problem of optimal protection from impact.

SYSTEM MODELING AND PROBLEM STATEMENT

The models used for computational modeling of automobile crashworthiness can be classified as lumped-parameter models, rigid multi-body (RMB) models, finite element (FE) models, and integrated models [5]. These models provide different levels of abstraction of the problem, and thus have different applications. In frontal impact, the occupant can be subject to injuries to various regions of the body, such as the head, thorax, upper extremities, and lower extremities. The injury criteria for measuring these injuries are usually expressed by impact responses (such as accelerations) in respective regions. When the prevention and reduction of injuries to the occupant are considered in general, the occupant can be reasonably treated as a point mass and its peak acceleration (deceleration) can be used as the injury criterion. When the vehicle crash pulse is under investigation, the vehicle is treated as a whole body and only its gross motion is considered.

Therefore, automobile frontal impact is described by a lumped-parameter model shown in Fig. 1, where the occupant is represented by a point mass m, $x(t)$ describes the gross motion of the vehicle, and $y(t)$ is the motion of the occupant relative to the vehicle. The interaction between the occupant and restraint systems, such as seat belt and airbag, is simply represented by a spring and a damper, but, in general, it can be described by a control force $u(y, \dot{y}, t)$. For the vehicle motion,

$$a(t) = -\ddot{x}(t),\qquad(1)$$

usually referred to as the vehicle crash pulse, and

$$D_v = \max_t \{x(t)\},\qquad(2)$$

defined as the vehicle crash deformation. The free space between the occupant and the vehicle interior is called the rattlespace S_0, which is the maximum allowable space for the occupant to move forward with respect to the vehicle, i.e.,

$$\max_t \{y(t)\} \le S_0.\qquad(3)$$

Since this space has important effects on the occupant response also and is a major factor in the restraint system design, it will be considered in addition to the crash deformation.
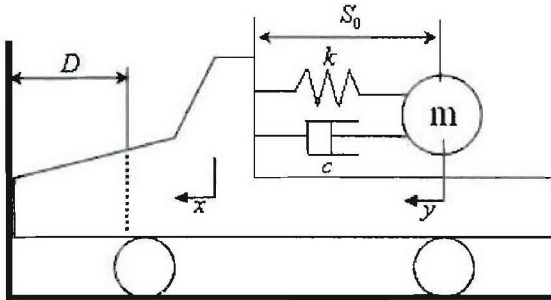

Figure 1. A lumped-parameter model of the system

The equation of motion of the system is

$$m(\ddot{x} + \ddot{y}) + u(y, \dot{y}, t) = 0,\qquad(4)$$

with the initial conditions

$$x(0) = 0,\ \dot{x}(0) = v_0,\qquad(5)$$

and

$$y(0) = 0,\ \dot{y}(0) = 0,\qquad(6)$$

where v_0 is the impact velocity.

The question to be addressed in this paper can be expressed as:

Problem-1: For given restraint characteristics and prescribed impact velocity and crash deformation, find the optimal crash pulse such that the peak occupant deceleration is minimized. The problem can be formulated as: Find the optimal vehicle crash pulse $a_0(t)$, such that

$$J_1(a_0) = \min_a \{J_1(a) \mid J_2(a) \le D_v, J_3(a) \le S_0\},\qquad(7)$$

where

$$J_1 = \max_t \{-[\ddot{x}(t) + \ddot{y}(t)]\},\qquad(8)$$

the peak occupant deceleration,

$$J_2 = \max_t \{x(t)\},\qquad(9)$$

the maximum vehicle displacement, and

$$J_3 = \max_t \{y(t)\},\qquad(10)$$

the maximum forward excursion of the occupant in the vehicle.

From the standpoint of the kinematics of the occupant, *Problem-1* can be reduced to a more generic problem:

Problem-2: For prescribed impact velocity, crash deformation, and rattlespace, find the optimal kinematics of the occupant such that the peak occupant deceleration is minimized while the maximum occupant forward displacement is bounded. The problem can be formulated as: Find the optimal kinematics of the occupant $w_0(t)$, such that

$$J_1(w_0) = \min_w \{J_1(w) \mid J_4(w) \le D_o\},\qquad(11)$$

where

$$J_4 = \max_t \{x(t) + y(t)\},\qquad(12)$$

which is the maximum forward displacement of the occupant, and

$$D_o = D_v + S_0,\qquad(13)$$

the allowable maximum forward excursion for the occupant in an inertial frame. Note that, unlike $a(t)$ in *Problem-1*, $w(t)$ is not a control function of the system. Instead, it is controlled or produced by $a(t)$ or $u(y, \dot{y}, t)$.

OPTIMAL OCCUPANT KINEMATICS

To find the optimal kinematics of the occupant for *Problem-2*, according to duality or reciprocity of

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optimization [6], the dual or reciprocal problem of *Problem-2* can be formulated as follows.

Problem-3: For prescribed impact velocity, vehicle crash deformation, and rattlespace, find the optimal kinematics of the occupant such that the maximum occupant displacement is minimized while the peak occupant deceleration is bounded. The problem can be formulated as: Find the optimal kinematics of the occupant $w_0(t)$, such that

$$J_4(w_0) = \min_w \{ J_4(w) \mid J_1(w) \le A_m \},\tag{14}$$

where A_m is the upper bound on the peak occupant deceleration.

To find the theoretical solution of *Problem-3*, denote

$$z(t) = x(t) + y(t),\tag{15}$$

which is the absolute motion of the occupant with respect to an inertial frame during impact. Consider the motion of the system from the initial of impact $t = 0$ to the instant $t = T_0$ when the occupant comes to a rest for the first time. That is,

$$v_0 + \int_0^{T_0} \ddot{z}(\tau)d\tau = 0.\tag{16}$$

The velocity of the occupant $\dot{z}(t)$ starts with v_0 at $t = 0$ and decreases to 0 at $t = T_0$, i.e.,

$$\dot{z}(t) \ge 0,\ 0 \le t \le T_0.\tag{17}$$

Therefore, for $0 \le t \le T_0$, the displacement of the occupant $z(t)$, which is given by

$$z(t) = \int_0^t \dot{z}(\tau)d\tau,\tag{18}$$

increases monotonically with respect to time and reaches its maximum value at $t = T_0$, that is,

$$\max_t \{ z(t) \} = z(T_0).\tag{19}$$

To minimize the maximum occupant forward displacement J_4,

$$J_4 = \max_t \{ x(t) + y(t) \} = \max_{t \in [0,T_0]} \{ z(t) \} = z(T_0),\tag{20}$$

T_0 should be minimized. According to the definition of T_0 (Eq. (16)), if the deceleration of the occupant $-\ddot{z}(t)$ takes the maximum allowable value A_m, that is,

$$\ddot{z}(t) = -A_m,\tag{21}$$

T_0 is minimized, and

$$T_0 = \frac{v_0}{A_m}.\tag{22}$$

This means that in order to minimize the peak displacement of the occupant, the deceleration of the occupant should remain constant at the value of A_m.

This is the optimal kinematics of the occupant for *Problem-3*.

Based on the duality or reciprocity between *Problem-2* and *Problem-3*, the optimal kinematics of the occupant for *Problem-2* can be stated as: In order to minimize the peak deceleration of the occupant, the deceleration of the occupant should remain constant at a value, which is denoted as A_m and is given by

$$A_m = v_0^2 / (2D_0).\tag{23}$$

This is also the optimal kinematics of the occupant for *Problem-1*.

THEORETICAL SOLUTION OF OPTIMAL CRASH PULSE

Linear Restraint Characteristics

Optimal Crash Pulse
Suppose the onset of the vehicle impact is at $t = 0$. Consider the motion of the occupant right after the onset of impact ($t = 0^+$) until a stop for the first time ($t = T_0$). As the initial conditions of the occupant are

$$z(0^+) = 0,\ \dot{z}(0^+) = v_0,\tag{24}$$

the optimal kinematics of the occupant is

$$\ddot{z}(t) = -A_m$$
$$\dot{z}(t) = v_0 - A_m t,\qquad 0^+ \le t \le T_0,\tag{25}$$
$$z(t) = v_0 t - \tfrac{1}{2} A_m t^2$$

where A_m is yet to be determined. From Eq. (4)

$$u(y, \dot{y}, t) = mA_m.\tag{26}$$

Suppose the characteristics of a restraint system can be expressed by a linear spring and a linear damper. That is,

$$u(y, \dot{y}, t) = ky + c\dot{y}.\tag{27}$$

Then,

$$ky + c\dot{y} = mA_m,\tag{28}$$

with initial conditions

$$y(0^+) = 0,\ \dot{y}(0^+) = 0.\tag{29}$$

The solution of Eq. (28) is

$$y(t) = \frac{mA_m}{k}(1 - e^{-\frac{k}{c}t})$$
$$\dot{y}(t) = \frac{mA_m}{c} e^{-\frac{k}{c}t},\qquad 0^+ \le t \le T_0.\tag{30}$$
$$\ddot{y}(t) = -\frac{kmA_m}{c^2} e^{-\frac{k}{c}t}$$

From Eqs. (15) and (25), the optimal motion of the vehicle can be expressed as

$$\ddot{x}(t) = -A_m(1 - \frac{km}{c^2}e^{-\frac{k}{c}t}), \tag{31a}$$

$$\dot{x}(t) = v_0 - A_m(t + \frac{m}{c}e^{-\frac{k}{c}t}), \qquad 0^+ \le t \le T_0, \tag{31b}$$

$$x(t) = v_0 t - \frac{1}{2}A_m t^2 - \frac{mA_m}{k}(1 - e^{-\frac{k}{c}t}). \tag{31c}$$

It follows that

$$x(0^+) = 0, \quad \dot{x}(0^+) = v_0 - A_m m/c. \tag{32}$$

Compare to Eq. (5), from $t = 0$ to $t = 0^+$, the velocity change of the vehicle is

$$\Delta v = x(0) - x(0^+) = \frac{m}{c}A_m. \tag{33}$$

To produce this velocity change, the acceleration of the vehicle needs to have an impulse I_x at the initial of impact $(t = 0)$. Theoretically, this impulse can be represented by a Dirac delta function $\delta(t)$; that is,

$$I_x(t) = \frac{A_m m}{c}\delta(t), \tag{34}$$

where $\delta(t)$ is given by

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \ne 0 \end{cases}, \tag{35}$$

and

$$\int_{-\infty}^{\infty}\delta(t)dt = 1. \tag{36}$$

As such, from the onset of impact $(t = 0)$ to the first stop $(t = T_0)$, the optimal vehicle acceleration is

$$\ddot{x}(t) = -A_m[1 + \frac{m}{c}\delta(t) - \frac{km}{c^2}e^{-\frac{k}{c}t}], \quad 0 \le t \le T_0. \tag{37}$$

Since impulse $I_x(t)$ results in only velocity change but no displacement change of the vehicle, Eq. (31c) still holds for $0 \le t \le T_0$.

Note that the exponential terms in Eqs. (30), (31), and (37) diminish with time. After a sufficiently long time they can be neglected.

Determination of A_m

If restraint characteristics are such that the maximum forward excursion of the occupant relative to the vehicle reaches the prescribed rattlespace, in other words, if the rattlespace is fully used, according to Eq. (23),

$$A_m = \frac{v_0^2}{2(D_v + S_0)}. \tag{38}$$

It follows from Eq. (31c) that

$$\max_t\{x(t)\} = x(T_0) \approx D_v + S_0 - \frac{mA_m}{k} = D_v, \tag{39}$$

where the exponential term is neglected. Then,

$$S_0 = \frac{mA_m}{k} = \frac{A_m}{\omega^2}, \tag{40}$$

where

$$\omega^2 = \frac{k}{m}, \tag{41}$$

which can be considered as the natural frequency of the restraint-occupant system. From Eqs. (38) and (40)

$$\omega^2 = \frac{v_0^2}{2S_0(D_v + S_0)}, \tag{42}$$

which represents a requirement for the restraint characteristics. This means that when the natural frequency of the restraint-occupant system satisfies Eq. (42), the rattlespace S_0 will be fully used for the occupant's excursion with respect to the vehicle.

In general, however, restraint systems are designed such that

$$\max_t\{y(t)\} < S_0, \tag{43}$$

i.e., the rattlespace is not fully used. In this case, it follows from Eqs. (23) and (31c) that

$$\max_t\{x(t)\} = x(T_0) \approx \frac{v_0^2}{2A_m} - \frac{mA_m}{k} = D_v, \tag{44}$$

where the exponential term in Eq. (31c) is neglected also. From Eq. (44),

$$A_m = \frac{kD_v}{2m}\left(\sqrt{1 + \frac{2mv_0^2}{kD_v^2}} - 1\right)$$
$$= \frac{\omega^2 D_v}{2}\left(\sqrt{1 + \frac{2v_0^2}{\omega^2 D_v^2}} - 1\right), \tag{45}$$

which indicates that the peak occupant deceleration or the amplitude of constant deceleration depends upon the impact speed, vehicle crash deformation, and the natural frequency of the restraint-occupant system.

According to Eqs. (30) and (45), Eq. (43) means that

$$\omega^2 > \frac{v_0^2}{2S_0(D + S_0)}, \tag{46}$$

which represents a requirement for the restraint characteristics such that the rattlespace is not fully used. It can be inferred that if

$$\omega^2 < \frac{v_0^2}{2S_0(D + S_0)}, \tag{47}$$

the excursion of the occupant will exceed the space limit. This is a situation not allowed to happen.

Illustration of Optimal Crash Pulse

From Eq. (37), the optimal crash pulse can be expressed as

$$A(t) = A_m[1 + \frac{m}{c}\delta(t) - \frac{km}{c^2}e^{-\frac{k}{c}t}]$$

$$= A_m[1 + \frac{1}{2\zeta\omega}\delta(t) - \frac{1}{4\zeta^2}e^{-\frac{\omega}{2\zeta}t}], \ 0 \le t \le T_0$$

(48)

where

$$\zeta = \frac{c}{2\omega m} = \frac{c}{2\sqrt{km}},$$

(49)

which is referred to as the damping ratio of the restraint-occupant system. Introducing ω and ζ reduces the parameters in Eq. (48) and avoids providing a value for m, which varies from occupant to occupant.

For the illustration of the optimal crash pulse, choose the values for the parameters in Eqs. (45) and (48) to be representative of automobile frontal impact [1]: $v_0 = 15.56$ m/s (35 mph), $D_v = 0.71$ m (28 in), $\omega^2 = 42.43$ rad/s, and $\zeta = 0.20$. The optimal crash pulse together with the velocity and displacement of the vehicle and the optimal kinematics of the occupant are illustrated in Fig. 2. Note that the impulse of the delta function at $t = 0$ should lead the deceleration of the vehicle at $t = 0$ to go infinite, but in the figure only a finite large value can be displayed. After $t = 0$, the acceleration of the vehicle jumps to a large positive value and then decays, and, after a certain period of time, it basically remains constant at a negative value. With this optimal crash pulse, the deceleration of the occupant remains constant during impact. After a certain time, the relative acceleration and velocity between the occupant and the vehicle become zero, and the relative displacement between them remains constant.

Trade-off Curve — A_m vs. S_0

For a prescribed optimal crash pulse (v_0 and D_v are given), if the natural frequency of a restraint system meets the requirement of Eq. (42), the given rattlespace S_0 will be fully used and then the peak occupant deceleration will be minimized. For the restraint system design, S_0 is an important factor and can be used as a common basis for the comparison of various designs. The relationship between the minimum peak occupant deceleration A_m and the prescribed rattlespace S_0 can be expressed in the form of a so-called trade-off curve. According to Eq. (38), the relationship between the peak occupant deceleration A_m and the rattlespace S_0 is shown in Fig. 3 by the blue curve for a range of S_0, where $v_0 = 15.56$ and $D_v = 0.71$. The curve indicates

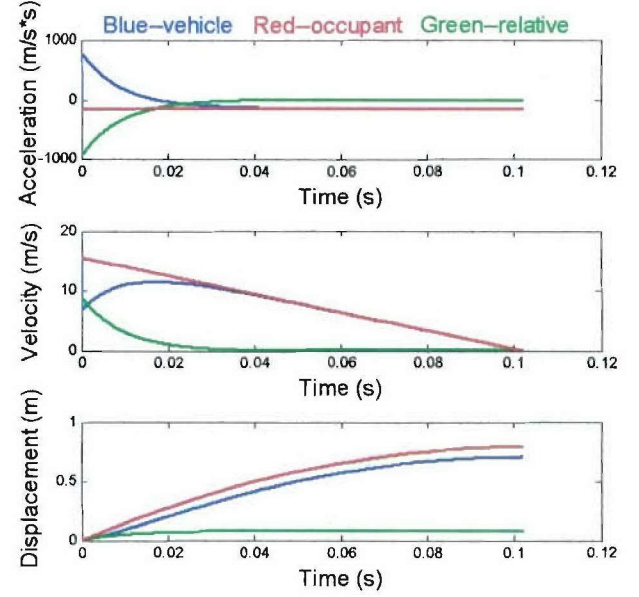that as the rattlespace S_0 increases, the peak occupant deceleration A_m decreases.
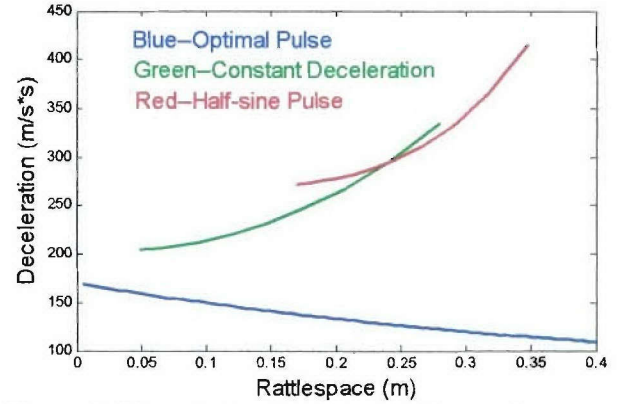

Figure 2. The motions of the vehicle and the occupant

Figure 3. The relationship between the peak occupant deceleration and the rattlespace

General Restraint Characteristics

In general, the characteristics of restraint systems can be so complicated that they cannot be reasonably described by a linear spring and a linear damper. However, regardless of the complexity of restraint characteristics, once they are given, the relative motion of the occupant with respect to the vehicle can be determined by solving Eq. (26), and then the optimal motion including the optimal crash pulse of the vehicle can be obtained. Whereas the optimal crash pulse depends upon the characteristics of particular restraint systems, certain general aspects of it can be ascertained based on the preceding analyses and results.

- If restraint systems are passive and have certain damping, an impulse is required at the initial of the optimal crash pulse.

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- If restraint systems are active and able to generate an impulsive action on the occupant actively, an initial impulse may not be required for the optimal crash pulse.
- The major portion of the optimal crash pulse after an initial period tends to be flattened.

NUMERICAL SOLUTION OF OPTIMAL CRASH PULSE

When restraint characteristics are nonlinear, or when restraint characteristics are linear but the motion of the vehicle is subject to other constraints such as a limit on the amplitude of the deceleration, the optimal crash pulse cannot be readily obtained analytically. Instead, numerical optimization can be utilized to find solutions.

In order to use numerical optimization methods to solve *Problem-1*, discretize the time interval $[0, T_0]$ into identical subintervals. In addition, represent the crash pulse $a(t)$ on each of the subintervals by a constant value, that is,

$$a(t) = a_i \quad \text{for} \quad (1-i)\Delta t \le t < i\Delta t, i = 1,\ldots,N, \quad [50]$$

where N is the number of subintervals, Δt is the length of each subinterval, and $\Delta t = T_0 / N$. Denote

$$\mathbf{A} = [a_1 a_2 \ldots a_N]^T \qquad [51]$$

as the vector of design variables. The motion of the occupant relative to the vehicle is discretized accordingly

$$y_i = y(i\Delta t), \ddot{y}_i = \ddot{y}(i\Delta t), i = 1,\ldots,N, \qquad [52]$$

and can be obtained from the numerical integration of Eq. (4). The system performance criteria of Eqs. (8) - (10) can be expressed by the discretized system responses:

$$J_1(\mathbf{A}) = \max_{1 \le i \le N}\{-(\ddot{x}_i + \ddot{y}_i)\}, \qquad (53)$$

$$J_2(\mathbf{A}) = \max_{1 \le i \le N}\{x_i\}, \qquad (54)$$

and

$$J_3(\mathbf{A}) = \max_{1 \le i \le N}\{y_i\}. \qquad (55)$$

Then *Problem-1* can be formulated as a numerical optimization problem:

Design Variables: $\mathbf{A}$;

Objective Function: $\min\{J_1(\mathbf{A})\}$; $\qquad (56)$

Constraints: $J_2(\mathbf{A}) \le D$

$$J_3(\mathbf{A}) \le S_0;$$

$$\mathbf{A}_L \le \mathbf{A} \le \mathbf{A}_U$$

where $\mathbf{A}_L$ and $\mathbf{A}_U$ are the lower and upper bounds on the crash pulse. A nonlinear optimization method can be used to solve this problem [7].

OTHER CRASH PULSES

The optimal crash pulse given by Eq. (48) is meaningful only in a theoretical sense, since the initial impulse, which is represented by $\delta(t)$, cannot be produced in practice. To be more practical, the vehicle deceleration during impact can be assumed to remain constant; that is, the crash pulse is a constant-deceleration pulse. More often, the vehicle deceleration in frontal impact is represented by a half-sine pulse. What are the occupant responses under these two "non-optimal" crash pulses? The issue will be investigated in the following analyses. Again, a restraint system is assumed to have linear characteristics, as described by Eq. (27).

It can be reasonably assumed that the peak occupant deceleration occurs within the duration of the vehicle crash pulse. Therefore, the occupant responses will be considered within the time span of the crash pulse in the following analyses. In the following computations, choose impact velocity $v_0 = 15.56$ m/s and crash deformation $D_v = 0.71$ m, to be representative of automobile frontal impact [1].

Constant Deceleration Pulse

Suppose the vehicle deceleration is constant, that is,

$$\ddot{x}(t) = -A_v, 0 \le t \le T_v, \qquad (57)$$

where

$$A_v = \frac{v_0^2}{2D}, \qquad (58)$$

and

$$T_v = \frac{2D_v}{v_0}, \qquad (59)$$

where T_v is the duration of the crash pulse. The velocity and displacement of the vehicle are

$$\dot{x}(t) = v_0 - A_v t, \qquad (60)$$

and

$$x(t) = v_0 t - \frac{1}{2}A_v t^2, \qquad (61)$$

From Eq. (28), the equation of motion of the occupant with respect to the vehicle is

$$\ddot{y} + 2\zeta\omega\dot{y} + \omega^2 y = A_v, \qquad (62)$$

with initial conditions

$$y(0) = 0, \dot{y}(0) = 0. \qquad (63)$$

By solving this equation, the motion of the occupant relative to the vehicle is given by

$$y(t) = y_c(t) + y_p(t). \qquad (64)$$

Here,

$$y_p(t) = \frac{A_v}{\omega^2}[1 - \frac{e^{-\zeta \omega t}}{\sqrt{1-\zeta^2}}\cos(\sqrt{1-\zeta^2}\omega t - \alpha_y)], \quad (65)$$

where

$$\alpha_y = \tan^{-1}(\frac{\zeta}{\sqrt{1-\zeta^2}}), \quad (66)$$

and

$$y_c(t) = A_c e^{-\zeta \omega t}\sin(\sqrt{1-\zeta^2}\omega t + \beta_y), \quad (67)$$

where

$$A_c = \sqrt{\frac{C_{1y}^2 + C_{2y}^2 + 2\zeta C_{1y}C_{2y}}{1-\zeta^2}}, \quad (68)$$

and

$$\beta_y = \tan^{-1}\frac{\sqrt{1-\zeta^2}C_{1y}}{\zeta C_{1y} + C_{2y}}. \quad (69)$$

In Eq. (68),

$$C_{1y} = \frac{A_v}{\omega^2}(\frac{1}{\sqrt{1-\zeta^2}}\cos\alpha_y - 1)$$

$$\quad (70)$$

$$C_{2y} = \frac{A_v}{\omega^2}(\sin\alpha_y - \frac{\zeta}{\sqrt{1-\zeta^2}}\cos\alpha_y)$$

The absolute motion of the occupant is the superposition of the vehicle motion and the relative motion:

$$z(t) = x(t) + y(t), \quad (71)$$

where $x(t)$ is given by Eq. (61) and $y(t)$ by Eq. (64).

The occupant responses under a constant-deceleration pulse are illustrated in Fig. 4, where $\omega = 35.43$ and $\zeta = 0.2$. It is shown that the occupant deceleration during impact is not constant, which means that the optimal kinematics of the occupant is not attained. In fact, under a constant-deceleration crash pulse, it is impossible for a passive restraint system to provide the required protection to the occupant such that the optimal kinematics of the occupant is achieved [8]. However, the restraint characteristics of a passive system can still be optimized to minimize the peak occupant deceleration as much as possible. Table 1 provides the values of the required rattlespace, optimal frequency, and minimal peak occupant deceleration corresponding to each level of damping for a linear passive restraint system, which were obtained from numerical optimization [8].

Based on the results given in Table 1, the relationship between the peak occupant deceleration and the rattlespace is displayed in Fig. 3 by the green curve. The curve indicates that the peak occupant deceleration increases with the increase of the rattlespace, which seems not logical. The consequence is due to the damping. In the range of [0.01 0.728], larger damping results in smaller required rattlespace and lower peak occupant deceleration. However, as shown in Fig. 3, for a given rattlespace, the peak occupant deceleration

produced by a constant-deceleration pulse is much larger than that resulting from the optimal crash pulse. The comparison of the peak occupant decelerations between the optimal crash pulse and the constant deceleration pulse is shown in Table 2 for a series of specific rattlespaces.
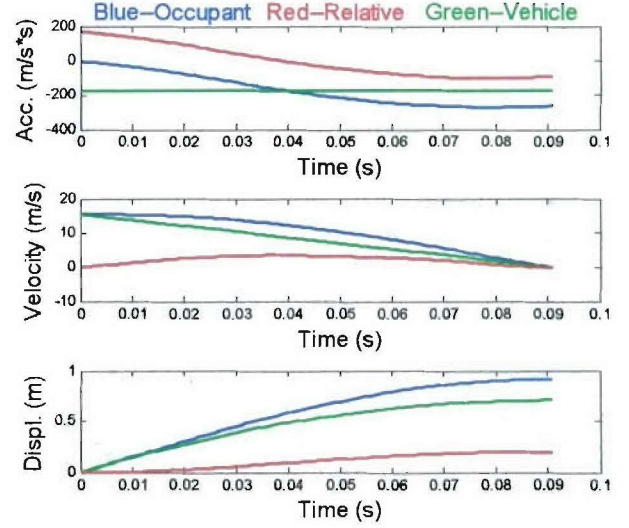

Figure 4. The motions of a system under a constant-deceleration pulse

Table 1. Optimization results for a constant deceleration pulse

ζ	ω (rad/s)	A_m (m/s^2)	S_0 (m)
0.01	34.63	335.76	0.280
0.10	34.83	297.37	0.243
0.20	35.43	267.99	0.207
0.30	36.47	247.39	0.176
0.40	38.10	232.48	0.147
0.50	40.60	221.38	0.120
0.60	44.61	212.93	0.094
0.70	52.73	206.35	0.064
0.728	60.39	204.77	0.048

Table 2. Comparison of peak occupant decelerations between optimal pulse and constant-deceleration pulse

S_0 (m)	A_{mo} (m/s^2) (Optimal Pulse)	A_{mc} (m/s^2) (Constant Deceleration)	$\frac{A_{mc} - A_{mo}}{A_{mo}} \times 100\%$
0.048	159.71	204.77	28.2
0.064	156.40	206.35	31.9
0.094	150.57	212.93	41.4
0.120	145.85	221.38	51.8
0.147	141.26	232.48	64.6
0.176	136.63	247.39	81.1
0.207	132.01	267.99	103.0
0.243	127.03	297.37	134.1
0.280	122.28	335.76	174.6

Half-sine Pulse

Suppose the vehicle acceleration is described by a half-sine pulse,

$$\ddot{x}(t) = -A_v \sin(\frac{\pi t}{T_v}), \quad 0 \le t \le T_v, \tag{72}$$

where

$$A_v = \frac{\pi v_0^2}{4D_v}, \tag{73}$$

and T_v is given in Eq. (59). The velocity and displacement of the vehicle are

$$\dot{x}(t) = \left\{ \frac{v_0}{2} + \frac{A_v T_v}{\pi} \cos \frac{\pi t}{T_v}, \right. \tag{74}$$

and

$$x(t) = \left\{ \frac{v_0}{2} t + \frac{A_v T_v^2}{\pi^2} \sin \frac{\pi t}{T_v}. \right. \tag{75}$$

It follows from Eqs. (4), (15), and (27) that

$$\ddot{z} + 2\zeta \dot{z} + \omega^2 z = \frac{v_0}{2}(2\zeta \omega + \omega^2 t) +$$
$$\frac{A_v T_v}{\pi}[2\zeta \omega \cos \frac{\pi t}{T_v} + \omega^2 \frac{T_v}{\pi} \sin \frac{\pi t}{T_v}] \tag{76}$$

Denote

$$\omega_v = \frac{\pi}{T_v}, \tag{77}$$

and

$$\alpha_z = \tan^{-1}(\frac{2\zeta \omega_v}{\omega}). \tag{78}$$

Then, Eq. (76) becomes

$$\ddot{z} + 2\zeta \omega \dot{z} + \omega^2 z = \frac{v_0}{2}(2\zeta \omega + \omega^2 t) +$$
$$\frac{\omega^2 A_v}{\omega_v^2} \sqrt{1 + (2\zeta \frac{\omega_v}{\omega})^2} \sin(\omega_v t + \alpha_z) \tag{79}$$

By solving this equation, the motion of the occupant under a half-sine pulse is found to be:

$$z(t) = z_c(t) + z_p(t). \tag{80}$$

where

$$z_c(t) = Z_c e^{-\zeta \omega t} \sin(\sqrt{1-\zeta^2} \omega t + \beta_z), \tag{81}$$

and

$$z_p(t) = \frac{v_0}{2} t +$$
$$\frac{A_v}{\omega_v^2} \sqrt{\frac{1+(2\zeta \frac{\omega_v}{\omega})^2}{[1-(\frac{\omega_v}{\omega})^2]^2 + (\frac{2\zeta \omega_v}{\omega})^2}} \sin(\omega_v t + \alpha_z - \varphi) \tag{82}$$

In Eq. (81),

$$Z_c = \sqrt{\frac{C_{1z}^2 \omega^2 + C_{2z}^2 + 2C_{1z}C_{2z}\zeta \omega}{(1-\zeta^2)\omega^2}}, \tag{83}$$

and

$$\beta_z = \tan^{-1}(\frac{C_{1z}\sqrt{1-\zeta^2} \omega}{C_{1z}\zeta \omega + C_{2z}}), \tag{84}$$

where

$$C_{1z} = -\frac{A_v}{\omega_v^2} \sqrt{\frac{1+(2\zeta \frac{\omega_v}{\omega})^2}{[1-(\frac{\omega_v}{\omega})^2]^2 + (\frac{2\zeta \omega_v}{\omega})^2}} \sin(\alpha_z - \varphi)$$

$$C_{2z} = \frac{v_0}{2} - \frac{A_v}{\omega_v^2} \sqrt{\frac{1+(2\zeta \frac{\omega_v}{\omega})^2}{[1-(\frac{\omega_v}{\omega})^2]^2 + (\frac{2\zeta \omega_v}{\omega})^2}} \cos(\alpha_z - \varphi) \tag{85}$$

In Eqs. (82) and (85),

$$\varphi = \tan^{-1} \frac{2\zeta \omega \omega_v}{\omega^2 - \omega_v^2}. \tag{86}$$

The occupant responses under a half-sine pulse are illustrated in Fig. 5, where $\omega = 32.78$ and $\zeta = 0.2$. Again, the optimal kinematics of the occupant is not attained. In fact, under a half-sine crash pulse, it is also impossible for a passive restraint system to provide the required protection to the occupant such that the optimal kinematics of the occupant is achieved [8]. However, the restraint characteristics can still be optimized to minimize the peak occupant deceleration as much as possible. Using numerical optimization [8], for a number of levels of damping, the values of the required rattlespace, optimal frequency, and the corresponding minimum peak occupant deceleration are listed in Table 3.
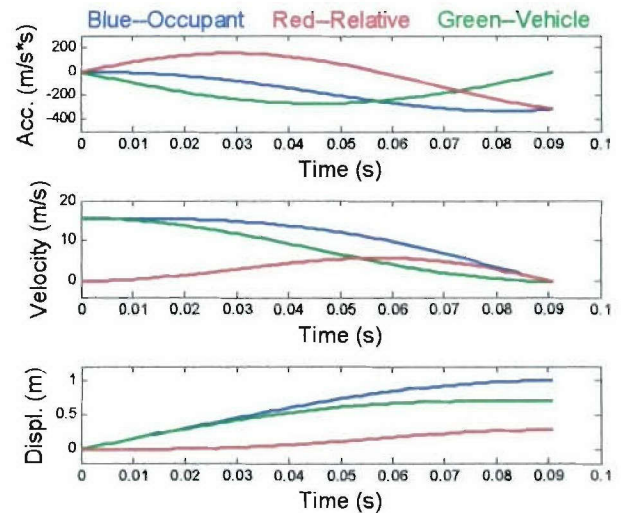

Figure 5. The motions of a system under a half-sine pulse

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Based on the results given in Table 3, the relationship between the peak occupant deceleration and the rattlespace is displayed in Fig. 3 by the red curve. The curve indicates that the peak occupant deceleration increases with the increase of the rattlespace, because the damping is controlled at each level. In the range of [0.01 0.99], larger damping results in smaller required rattlespace and lower peak occupant deceleration. It can be seen from Fig. 3 among the three crash pulses considered, the optimal crash pulse is the best, followed by the constant-deceleration pulse, whereas the half-sine pulse is the worst, in terms of the resulting peak occupant deceleration. The comparison of the peak occupant decelerations between the optimal crash pulse and the half-sine pulse is shown in Table 4 for a series of specific rattlespaces.

Table 3. Optimization results for half-sine pulse

ζ	ω (rad/s)	A_m (m/s^2)	S_0 (m)
0.01	34.51	414.76	0.348
0.10	33.62	365.16	0.317
0.20	32.78	331.12	0.289
0.30	32.06	310.20	0.265
0.40	31.44	297.02	0.245
0.50	30.91	288.46	0.228
0.60	30.44	282.75	0.213
0.70	30.04	278.87	0.20
0.80	29.68	276.18	0.188
0.90	29.37	274.24	0.178
0.99	29.11	272.96	0.169

Table 4. Comparison of peak occupant decelerations between optimal pulse and half-sine pulse

S_0 (m)	A_{mo} (m/s^2) (Optimal Pulse)	A_{ms} (m/s^2) (Half-sine Pulse)	$\dfrac{A_{ms} - A_{mo}}{A_{mo}} \times 100\%$
0.169	137.72	272.96	98.2
0.178	136.33	274.24	101.2
0.188	134.81	276.18	104.9
0.20	133.03	278.87	109.6
0.213	131.16	282.75	115.6
0.228	129.06	288.46	123.5
0.245	126.76	297.02	134.3
0.265	124.16	310.20	149.8
0.289	121.18	331.12	173.3
0.317	117.87	365.16	209.8
0.348	114.42	414.76	262.5

CONCLUDING REMARKS

In automobile frontal impact, the optimal kinematics of the occupant is such that the occupant moves at a constant deceleration. For given restraint characteristics, it is possible for the occupant to attain optimal kinematics by optimizing the vehicle motion or the vehicle crash pulse. For linear restraint characteristics, the optimal vehicle acceleration should have a large negative impulse at the initial of impact;

after that, it jumps to a large positive value and then decays; and after a certain period of time, it basically remains constant at a negative value. In general, the optimal crash pulse depends upon the characteristics of particular restraint systems.

Different crash pulses have different influences on the occupant motion and the restraint system design. For linear restraint characteristics, the peak occupant deceleration resulting from the optimal crash pulse is much lower than those produced by a constant-deceleration pulse or a half-sine pulse. Under the optimal crash pulse, the peak deceleration is independent of the damping of restraint systems. However, certain damping in the systems is necessary, and large damping can reduce the amplitude of the initial impulse. Under a constant-deceleration pulse or a half-sine pulse, the damping of restraint characteristics is important and determines the required rattlespace and natural frequency. In a certain range of the damping, the peak occupant deceleration decreases with the increase of the damping.

It is recognized that the use of a lumped-parameter model for the occupant-vehicle system and the assumption of linear characteristics for restraint systems may have imposed certain limitations on the analyses in this paper. However, the results and conclusions derived can still be used as general guidelines for the crashworthiness design of the vehicle structure.

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